TOPOLOGY-IV
M.MATH-II
MID-TERM EXAM

Total 30 Marks

(1) Show that $M \times N$ is orientable if and only if M and N are orientable. (4 marks)

(2) Show that TM is orientable as a smooth manifold. (5 marks)

(3) Let S^n be the standard unit sphere in $\mathbb{R}^{n+1}$.

(a) Give a no-where vanishing section of the tangent bundle $T(S^3)$. (2 marks)

(b) Show that $T(S^n) \oplus \epsilon^1$ is a trivial bundle, where ϵ^1 is a trivial bundle of rank 1. (4 marks)

(c) Show that $T(S^3 \times S^n)$ is a trivial bundle. (5 marks)

(4) Let W and Y are compact and oriented manifolds and let $k = \dim Y = \dim W - 1$. If $X = \partial W$ and $f : X \to Y$ extends smoothly to all of W, then

$$\int_X f^*\omega = 0$$

for every k-form ω on Y. (5 marks)

(5) Let N be a domain with boundary in a m-dimensional compact oriented manifold M. Show that the map

$$\int_{\partial N} : H^{m-1}(M) \to \mathbb{R}$$

is a zero map. (5 marks)